UK INTERMEDIATE MATHEMATICAL CHALLENGE

THURSDAY 3RD FEBRUARY 2000

Organised by the **United Kingdom Mathematics Trust from the School of Mathematics, University of Leeds**

SOLUTIONS LEAFLET

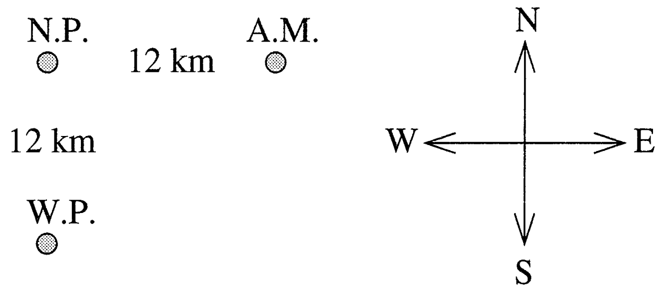
This solutions leaflet for the IMC is sent in the hope that it might provide all concerned with some alternative solutions to the ones they have obtained. It is not intended to be definitive. The organisers would be very pleased to receive alternatives created by candidates.

1. **B** Take the units digits of any two numbers and multiply them together. The units digit of the answer is also the units digit of the product of the original two numbers. As $7 \times 9 = 63$, the units digit of 567×3489 must also be 3.

2. **A** The angle occupied by the 'chocolate' sector is $\frac{1}{2}(360° - 90°) = 135°$. This is $1\frac{1}{2}$ times bigger than the 'strawberry' sector and hence the number of chocolate ice creams sold is $1\frac{1}{2} \times 60 = 90$.

3. **E** $\frac{6}{11}$ is the only one of these fractions which is greater than $\frac{1}{2}$.

4. **C**



5. **C** The numbers along the leading diagonal total 58 and this is therefore the sum of each row and column. We can now calculate that the number to the left of the '10' must be 20 and the number below that is 7.
Hence $x = 58 - (16 + 14 + 7) = 21$.

(A more difficult task is to calculate the value of the number in the top right-hand corner of the magic square. Can you do this?)

6. **B** Granny dropped 4 cups and 5 saucers, leaving her with 8 cups and 5 saucers. Therefore 3 cups did not have matching saucers.

7. **D** $5^2 + 2 = 3^3$

(This is in fact the only solution of this equation for which x and y are positive whole numbers. Another way of looking at this is to say that 26 is the only whole number which is 'sandwiched' between a perfect square and a perfect cube. This was first proved by the French mathematician, Pierre de Fermat, in the 17th century.)

8. **C** A, B and C are all equidistant from D and therefore lie on a circle whose centre is D. BC is a diameter of the circle and $\angle BAC$ is therefore the angle subtended by a diameter at a point on the circumference (the angle in a semicircle).

(Alternatively: let $\angle ACD = x$ and show that $\angle DAC = x$, $\angle ADB = 2x$ and $\angle DAB = \frac{1}{2}(180° - 2x) = 90° - x$. Hence $\angle BAC = x + 90° - x = 90°$.)

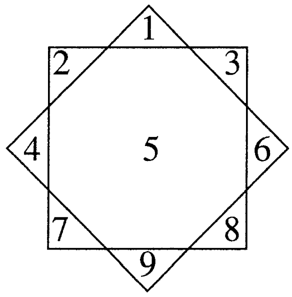
9. **B** The numbers of multiples of 4 between 2001 and 3001 is 250. However, the following years will not be leap years: 2100, 2200, 2300, 2500, 2600, 2700, 2900, 3000. This leaves 242 leap years.

10. **D** The total weight of the original five dates was 250g and the total weight of the four remaining dates was 160g.

11. D The sale price is 75% of the original price. Therefore the amount I saved, 25% of the original price, is one third of £240, i.e. £80.

12. E Timmy takes 24 minutes (8/20 of 1 hour) to reach the surgery; Tammy takes 18 minutes (12/40 of 1 hour) and Tommy takes 22 minutes (33/90 of 1 hour). The order, therefore, is Tammy, Tommy, Timmy.

13. B



14. D $\frac{b}{c} = \frac{a}{c} \times \frac{b}{a} = \frac{3}{4} \times \frac{3}{2} = \frac{9}{8}$.

15. C There are 9 such numbers whose first digit is 1 : 110, 121, 132, …, 187, 198. Similarly there are 8 such numbers, beginning with 220 and ending with 297, whose first digit is 2; 7 such numbers, beginning with 330 and ending with 396, whose first digit is 3 and so on. Lastly there is only 1 such number whose first digit is 9: 990.

The answer, therefore, is $9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 45$.

16. A The number may be divided up into 400 blocks of '12345'. The sum of the digits in each block is 15 and hence the sum of all 2000 digits is $400 \times 15 = 6000$. Alternatively, the mean of the digits which make up the number is 3 and therefore the sum of the digits is $2000 \times 3 = 6000$.

17. D Buying two more parsnips and three fewer turnips does not change the total cost and hence two parsnips cost the same as three turnips. Instead of six parsnips, therefore, Baldrick could have bought nine turnips and, together with seven turnips, this makes a total of sixteen turnips. Alternatively, he could have bought twelve turnips instead of the eight parsnips and, together with four turnips, this makes sixteen turnips.

18. D We need to write $3^4 \times 4^5 \times 5^6$ in the form $a \times 10^n$ where a is not a multiple of 10. $3^4 \times 4^5 \times 5^6 = 3^4 \times 2^{10} \times 5^6 = 3^4 \times 2^4 \times 2^6 \times 5^6 = 3^4 \times 2^4 \times 10^6$. Hence the number ends in six zeros.

19. C We need to express 1800 as the product of two factors, one of which (her age in months) is between twelve and thirteen times the other (her age in complete years). These are 150 and 12 respectively. Mary is 150 months old i.e. she was twelve on her last birthday and she is now 12 years 6 months old.

20. C The percentage increases are A 25%; B 40%; C $42\frac{6}{7}$%; D 30%; E $33\frac{1}{3}$%

21. E On the outside of the wire, the pencil describes an arc of a circle as the disc rolls around each of the corners of the triangle, but this does not happen when the disc moves around the inside of the wire.

22. C The smallest possible number of pairs of students with the same mark will occur when every possible mark from 0 to 100 is awarded to at least one student. This accounts for 101 students and therefore the remaining 19 students must all be awarded the same mark as exactly one of their colleagues. The 120 students are made up of 19 pairs of students who are awarded the same mark and 82 students who are all awarded a different mark from everyone else.

23. B The interior angle of a regular nine-sided polygon $= 180° - (360° \div 9) = 140°$. Consider the pentagon $ABCDE$: $\angle EAB = \frac{1}{2}(540° - 3 \times 140°) = 60°$. Similarly, $\angle FAI = 60°$ and hence $\angle FAE = 140° - (60° + 60°) = 20°$.

24. A Let the number of ivy, nightshade and triffid plants be i, n and t respectively. Then:

$2i + 9n + 12t = 120$ and $i + n + t = 20$, where $i > 0; n > 0; t > 0$. Multiplying the second equation by 2 and subtracting the new equation from the first:

$$7n + 10t = 80$$

$$\text{Thus } 7n = 10(8 - t)$$

Therefore n is a multiple of 10 and since $1 \leqslant n < 20, n = 10$.
Hence $8 - t = 7$ and therefore $t = 1$.

25. A Let the large circles have radius R.

Area $A = 2 \times \pi \times (\frac{1}{2}R)^2 = \frac{1}{2}\pi R^2 \approx 1.6R^2$.

Area $B = 3 \times \frac{1}{2} \times R \times R \times \sin 120° = \frac{3}{4}\sqrt{3}R^2 \approx 1.3R^2$.

Area $C = (2R)^2 - \pi R^2 = (4 - \pi)R^2 \approx 0.9R^2$.

Area $D = 2 \times \frac{1}{2} \times R^2 = R^2$.

Area $E = \pi R^2 - (\sqrt{2}R)^2 = (\pi - 2)R^2 \approx 1.1R^2$.